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Analysis of Stress-Strain Curves of Various Metals by Macroscopic Nonlinear-Defect-Model

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A nonlinear-defect-model was proposed previously by the author in order to explain the stress(σ)-strain(ε) curve by introducing a macroscopic defect with mechanically nonlinear character. In the present paper, the expression of the curve in the plastic region is modified experimentally into

$$\varepsilon(1-\beta\varepsilon)=B[1-b\times\exp(-\varepsilon/\varepsilon_0)]\sigma$$

where β, B, b and ε_0 are certain constants.

The above expression is applied for the different types of the curves and it was obtained that (i) the expression approximates the curves for steel, aluminium, copper and brass with the error less than 2%, (ii) b=1 stands for all the above metals with the error less than 5% and so on.

1. Introduction

Mechanical property of materials at the deformation or fracture is much complicated. Many reports have been given so far for individual interest. For example, (i) the stress-strain curve was analyzed[1] by using the empirical expression, (ii) the strength of the material was discussed[2] statistically based upon the distributed Griffith's defects with different yield strength. (iii) The microscopic or crystallographic study of the grain has been developed recently on the basis of the experimental and theoretical successes for the single crystal. (iv) Different clusters of defect are recently discussed such as optical dimer

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or trimer, magnetic spin glass and so on.

In order to explain macroscopically the generation of final fracture plane by a tensile force in relation to (i) and (ii), the author has proposed a nonlinear-defect-model (denoted by NDM) modifying the simple elastic theory. For example, the stress-strain curve was explained well by assuming a mechanically non-linear character of a macroscopic defect (not atomic one).

In the present paper it is shown that the improved NDM is well available for various metals.

2. Nonlinear-Defect-Model and Stress-Strain Curves

The NDM was proposed in order to explain the deformation of the matter and the generation of a final fracture surface and is quoted shortly as follows:

Let us assume that a mechanically nonlinear defect (ND) lies in a one-dimensional crystal and has Young's modulus E depending on the strain ε through $E=E_0(1-\beta\varepsilon)$, where β and E_0 are constants. Then the stress(σ)-strain curve and fracture stress σ_c of the ND are given by

$$\varepsilon(1-\beta\varepsilon)=(1/E_0)\sigma, \quad \sigma_c=E_0/4\beta \qquad (1)$$

It was shown also that ε tends to $1/\beta$ when the ND was fractured.

The NDM was improved[3,4] when various small NDs are distributed in a body, refering 1.(ii) and (iv). Then the stress-strain curve of the body was given from statistical view point (Appendix I), and was also approximated simply using the curve of large defects, that is, an equivalent-nonlinear-defect-approximation: ENDA (cf. Appendix II) is written by

$$\bar{\varepsilon}(1-\bar{\beta}\bar{\varepsilon})=\bar{A}\sigma \qquad (2)$$

where $\bar{\beta}$, $\bar{A}$ are certain parameters depending on $\bar{\varepsilon}$. It is noted that eq.(2) was discussed individually in the following three regions are reproduced in Fig.1.

(A1) Initial region R_i.... For a small strain, all the small defects F, distributed randomly in the body are not yet fractured and lead the macroscopic elastic strain.

(A2) Fracture region R_f.... In the final fracture, only the defect group,F*, distributed around a visual fracture surface are deformed or fractured, where the stress concentration factor p at the point of the macroscopic crack and the decrease of the cross section of the body are introduced.

(A3) Transient region R_t.... For a intermediate strain, the small defects under consideration must be concentrated from F to F* according to the elongation, so that $\bar{A}$ varies slowly according to $\bar{\varepsilon}$. The defect group F* may be a certain groups of the defects densely distributed among F (cf. Appendix).

It was obtained in the experiment that $\beta_t=\beta_f$ stands.

3. Improved Approximation

In the previous paper[3)] it was assumed firstly that the maximum stress point $P_c(\varepsilon_c,\sigma_c)$ or P_4 in Fig.1 belongs in R_f-region. Then $\beta_f=\varepsilon_c/2$ and $A_f=\varepsilon_c/2\sigma_c$ are obtained and the curve R_f is determined. The curve $A_t(\varepsilon)$,calculated from the observed curve and the R_f-one, can be approximated by $A_t=A_{t0}\varepsilon^{1/2}$(A_{t0} is a constant) so that the curve R_t is formulated.

Now the expression for $A_t(\varepsilon)$ should be improved from the experimetnal view point, where the bar on the latter in eq.(2) is omitted for the sake of simplicity and the notations in Fig.1 are used, as follows:

(a) Elastic deformation (the same with the previous method)... The final point $A(\varepsilon_i,\sigma_i)$ of the R_i-region gives $E_0=1/A_i=\sigma_i/\varepsilon_i$, $\beta_i=1/2\varepsilon_i$, where E_0 is defined for the observed stress-strain curve, but not for the infinitesimal strain.

(b) Plastic deformation.... It is doubtful that the point P_4 belongs in the R_f-region, because the visual cracks appear rather at the final stage of the curve. When the point $P_1 \sim P_6$ in Fig.1 belong to the R_f-region, respectively, the corresponding curves of $A(\varepsilon)$ are shown in Fig.2. The curves for P_5 and P_6 fit well

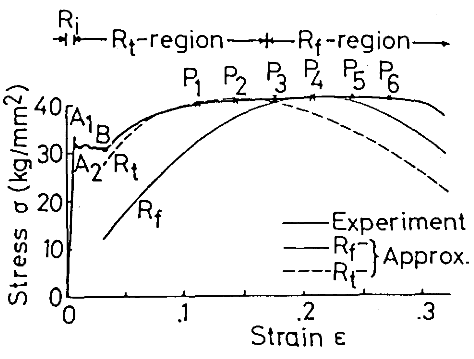

Fig.1 Approximation in R_i-, R_t- and R_f-regions

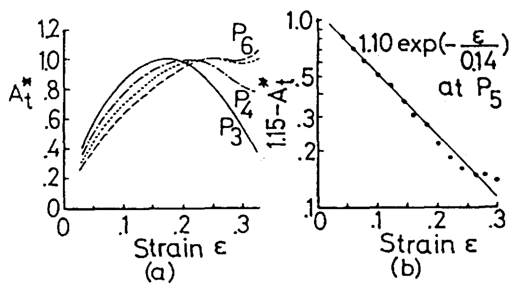

Fig.2 Estimation of A_t^*

with 2.(A_2) and (A_3), and are well expresssed by

$$\varepsilon(1-\beta\varepsilon)=B[1-b\times\exp(-\varepsilon/\varepsilon_0)]\sigma \qquad (3)$$

where β, B, b and ε_0 are constants and calculated by the following successive approximation method, denoting the n-th approximation by the suffix n:

(b1) First approximation... Let us denote the largest strain in experiment by ε_F and $\varepsilon_x=(\varepsilon_c+\varepsilon_F)/2$, and assume that the point $P(\varepsilon_x,\sigma_c)$ belongs in R_f-region (b=0). Then $\beta 1=\varepsilon_x/2$ and $B1=\varepsilon x/2\sigma x$ are obtained, so that the stress $\sigma_{t1}(\varepsilon)$ is defined.

(b2) Second approximation.... When certain strains ε_s and ε_1 satisfy $\varepsilon_s\ll\varepsilon_0\ll\varepsilon_1$, we have the approximated relations:

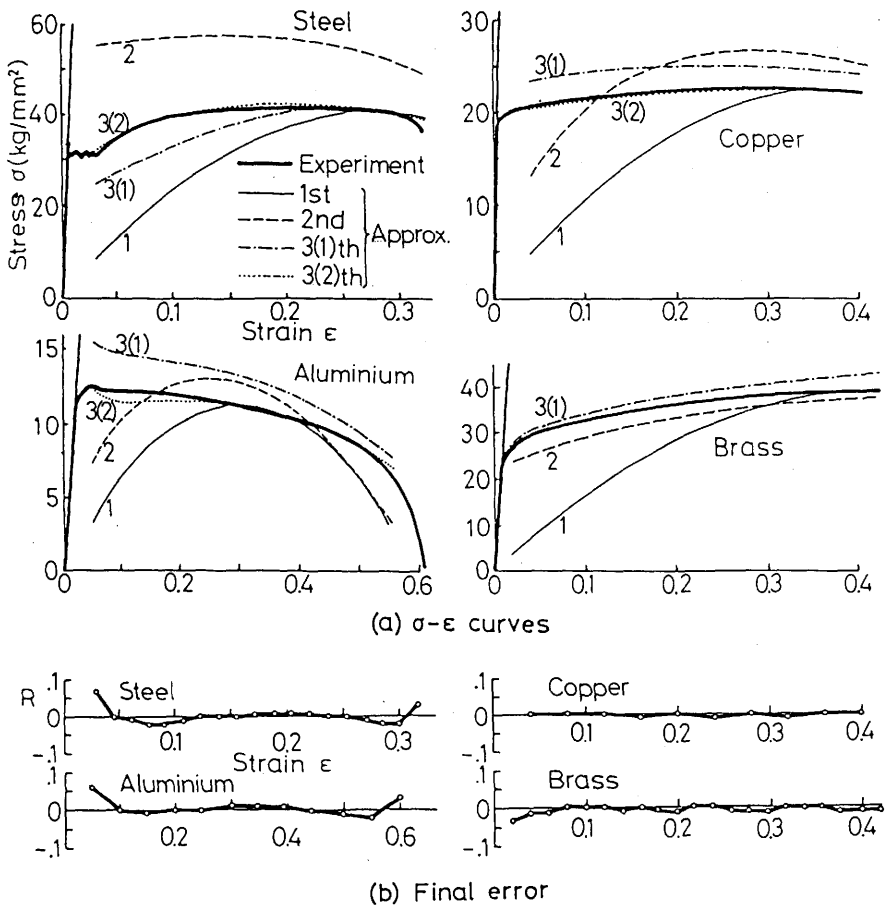

Fig.3 Approximation for curves of various metals

$$\varepsilon_{02}=(\varepsilon_1-\varepsilon_s)/\log 10,\quad B_2=B_1\sigma_{t1}(\varepsilon_1)/\sigma(\varepsilon_1),$$

$$b_2=[B_2-B_1\sigma_{t1}(\varepsilon_s)/\sigma(\varepsilon_s)]/B_1 \qquad (4)$$

(b3) Third approximation....When β_3, B_3, b_3 and ε_{03} are expressed as

$$\beta_3=\beta_1+\Delta\beta,\quad B_3=B_2+\Delta B,\quad b_3=b_2+\Delta b,\quad \varepsilon_{03}=\varepsilon_{02}+\Delta\varepsilon_0 \qquad (5)$$

where $\Delta\beta$, ΔB, Δb and $\Delta\varepsilon_0$ are small, the small quantities are approximately obtained by using the least square method. The approximation at the n times repetition of eq.(5) is denoted with the suffix 3(n) such as $\beta_{3(n)}$.

4. Approximation for Various Metals

The present method is applied for several commercial metals as shown in Fig.3. The curve for steel S22C has the maximum stress point at the mediate strain, and for aluminium at the much small strain. The curves for copper and brass are fairly flat. Considering of the experimental accuracy, the successive approximation (b3) is stopped when the maximum relative correction r(the maximum ones among $r_b=\Delta B/B$ and the others) becomes less than $r^*=0.01$. The following result is obtained:

(i) The relative error $R=(\sigma_{calc}/\sigma-1)$ in the final (b3) approximation is given in Fig.3(b). It is confirmed that the present method is much useful for various metals, and the constants obtained are compared in Table 1. We note that the observed curve may be in the appreciable error when ε is small, so that E_0 and β_1 also. The values of B, β, b and ε_0 are fairly reliable.

Table 1 Parameters for various metals, where E_0 is in 10^{-4}kg/cm^2 and B in 10^6cm^2/kg

Sample	Last approx.	Regions: Elastic E_0	β_1	Plastic β	B	b	$100\varepsilon_0$
Steel (S22C)	3(3)	247	272	1.32	56.5	0.95	28
Al	3(4)	57.6	250	13.4	22.4	1.04	2.4
Cu	3(3)	63.1	167	1.01	141	1.00	28
Brass	3(4)	246	50	0.99	74	0.99	22

(ii) It is found commonly in all the metal that (a) b=1 stands with the error less than 5% and (b) β is much smaller than β_i, so that the R_i- and R_t-regions should be governed by the remarkably different kinds of macroscopic ND. The latter result supports that (A1) and (A3) are corresponded microscopically to F and F*, respectively.

(iii) Aluminium has the remarkably remote values of β and ε_0.

Lastly we shall refer to that the stress-strain curve has been approximated by

$$\sigma^*=C\varepsilon_n \qquad (6)$$

for the experimental convenience, where σ^* is actual stress and C and n are constants. For a fairly great stress such that the considerable necking appears, eq.(6) is hardly available.

Comparing eqs.(3) and (6) in order to describe an observed σ-ε curve, eq.(6) requires three parameters: C, n and varying area in order to rewrite σ into σ^*. As b = 1 stands in Table 1, eq.(3) does also three parameters, whereas the eq.(3) is favorable because of the good fit with the curves of remarkably different types. The basis of b=1 is remained.

5. Conclusion

It was previously reported that the nonlinear-defect-model can approximate the stress-strain curves of steel. In the present paper the curves for various metals are discussed as follows:

(i) The plastic region of the stress-strain curve can be approximated by the improved expression:

$$\varepsilon(1-\beta\varepsilon)=B[1-b\times\exp(-\varepsilon/\varepsilon_0)]\sigma$$

where β, B, n and ε_0 are constants. It was confirmed that different types of the curves for steel, aluminium, copper and brass can be approximated with the error less than 2%.

(ii) Comparing the above constants obtained for various metals, it was found that b=1 stands commonly in all the metals and that only aluminium has remote values of β and ε_0.

(iii) It is noted that the remarkably different values of β are obtained for R_i- and R_t-regions, so that the kind of macroscopic NDs should be also. This fit to the previous discussions that the two regions depend on the different groups among the microscopic NDs.

Appendix

In the previous paper[3,4] the stress-strain curve was discussed in the three types of nonlinear-defect-approximation (NDA):

(i) Equivalent-NDA (ENDA).... The curve of the body is approximated by eq.(2) of an equivalent macroscopic ND. (ii) Single-NDA (SNDA).... A large single ND which is defined by eq.(1) is assumed on the visual fracture surface and leads eq.(2) for the body. (iii) Different-NDA (DNDA).... The curve should be governed by different kinds of small NDs distributed in the body.

Some of the results are shortly quoted here.

I. Residual Strain in DNDA

It is assumed that (i) m kinds of small defects are distributed in one-dimensional crystal of total atom number N, and n_i defects of i-th kind have the nonlinear parameter β_i and fracture stress σ_{ci}, where the defects are ranked in the order of small fracture stress. (ii) When the crystal is tensiled, lattice space between each two adjacent atoms varies in a linear elastic way when similar atoms appear and in ways similar to eq.(1) when the defect appears.

Then for example in R_t-region, the applied stress σ may lead the fracture of the defects of $1\sim j$-th kinds, so that also the strain:

$$\bar{\varepsilon}=\bar{\varepsilon}_e+\bar{\varepsilon}_s+\bar{\varepsilon}_{rj} \qquad \text{(A-1)}$$

where

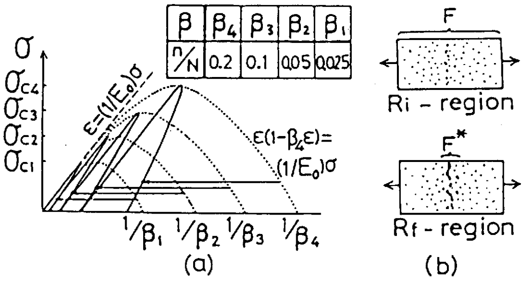

Fig.A-1 Dynamic stress-strain curve

Table A-1 Parameters for stress-strain curve

	Region	β	A
SNDA	Ri	β_i	$A_i = 1/E_0$
	Rt	$\beta_t(\varepsilon)$	$A_t(\varepsilon)$
	Rf	$\beta_f = a$	$A_f = p/E_0$
ENDA	Ri	$\bar{\beta}_i = 0$	$\bar{A}_i = (1+f)/E_0$
	Rt	$\bar{\beta}_t(\varepsilon)$	$\bar{A}_t(\varepsilon)$
	Rf	$\bar{\beta}_f = \frac{2\beta}{1+f}$	$\bar{A}_f = \frac{p(1+f)}{2E_0}$

$$\bar{\varepsilon}_e=(N-\sum_{i=1}^{m} n_i)\sigma/NE_0$$

$$\bar{\varepsilon}_s=\sum_{i=1}^{m}(n_i/-2\beta_i N)-\sum_{i=j+1}^{m}[(n_i/2\beta_i N)\sqrt{1-4\beta_i\sigma/E_0}]$$

$$\bar{\varepsilon}_{rj}=\sum_{i=1}^{j}(n_i/2\beta_i N)=\sum_{i=1}^{j}(4n_i\sigma_{ci}/NE_0)$$

and ε_{rj} is the residual strain. The present result gives the dynamic stress-strain curve illustrated schematically in Fig.A-1, whereas the quantitative discussion was remained.

It was noted also that (a) the present defects, called by the defect-colonies, should be micrographic clusters of small defects. (b) We observed in a KCl single crystals mechanical, optical or photoelastic defect-colonies[5,6] in close relation to the cleavage and (c) it was suggested that the planner distribution of defect-colonies may be concerned considerably at the spontaneous cleavages[7], but less at the artificial ones.

II. Stress-Strain Curve in ENDA and SNDA

Eq.(2) in ENDA was reduced from eq.(1) in SNDA as follows: (i) R_i-region.... If a single defect S with a nonlinear parameter β_s is located and has a relative length f in a crystal, so that the defect has the fracture stress $\sigma_s=E_0/4\beta_s$ and the corresponding strain $\varepsilon_s=1/2\beta_s$, we have the σ-ε curve of the body as

$$\bar{\varepsilon}_s=[(1+f)/E_0]\sigma_s \qquad \text{(A-3)}$$

(ii) R_f-region.... In a two-dimentional body, a visual crack of the length r leads the local stress σ_L at the point of the crack under the stress concentration factor p and the effective stress σ_e by

$$\sigma_L=p\sigma_e=p\sigma/(1-a\varepsilon)$$

where $\varepsilon=ar$ is assumed simply. This is the other type of the nonlinear defect with the nonlinear parameter $\beta_s=a$. Then eqs.(1) and (A-3) are displaced by

$$\varepsilon(1-\beta_s\varepsilon)-(p/E_0)\sigma \qquad \text{(A-4)}$$

$$\varepsilon[1+\{2\beta_s/(1+f)\}\varepsilon]=[p(1+f)/2E_0]\sigma \qquad \text{(A-5)}$$

The stress concentration will be analyzed in future in the view point of DNDA. (iii) Eqs.(1) and (A-5) are written by eq.(2) if the two parameters β and A vary slowly according to $\bar{\varepsilon}$. That is, R_i-region corresponds to only the term ε_e in eq.(A-1), R_t-region to the whole terms and R_f-region to those of j=m. In the view point of DNDA, the defects F and F* in 2.(A1) and (A2) may be as shown in Fig.A-1(b) and the stress concentration will be analyzed in future.

Then the relation shown in Table A-1 was obtained.

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